Tensoes Variadas e Concentracao de tensões

Elementos de Maquinas 1

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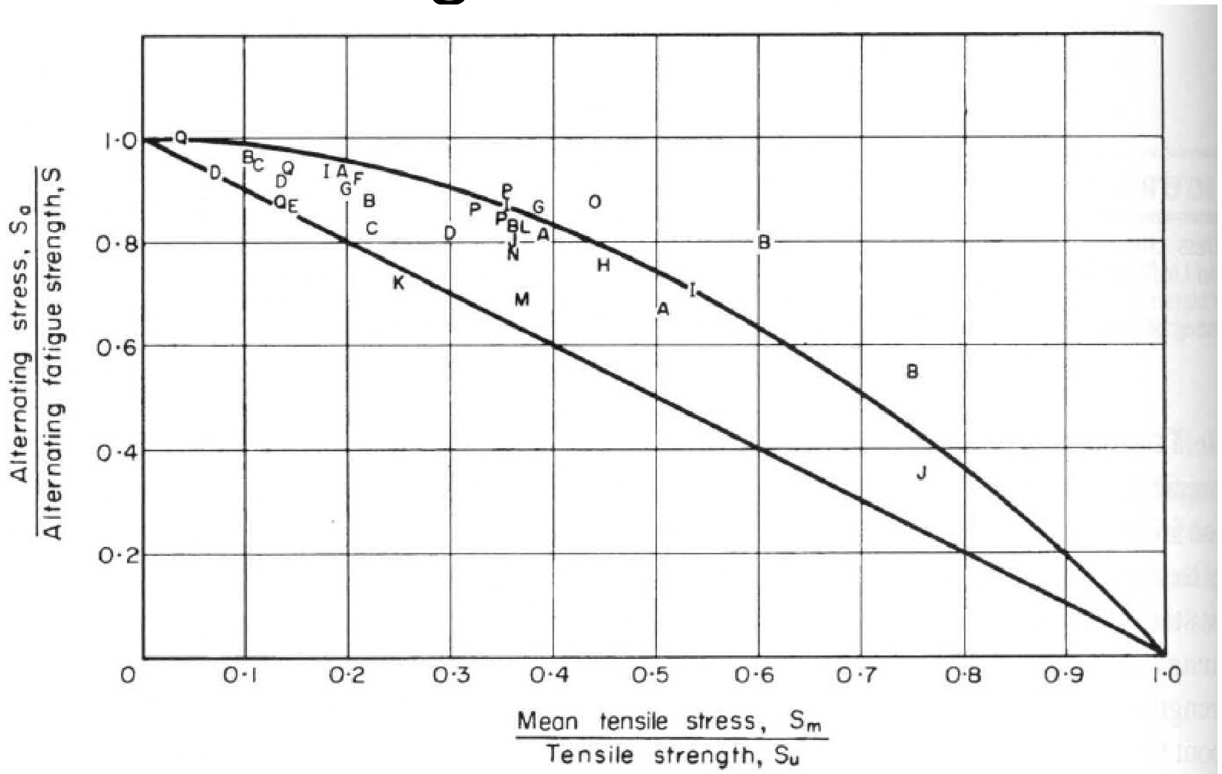
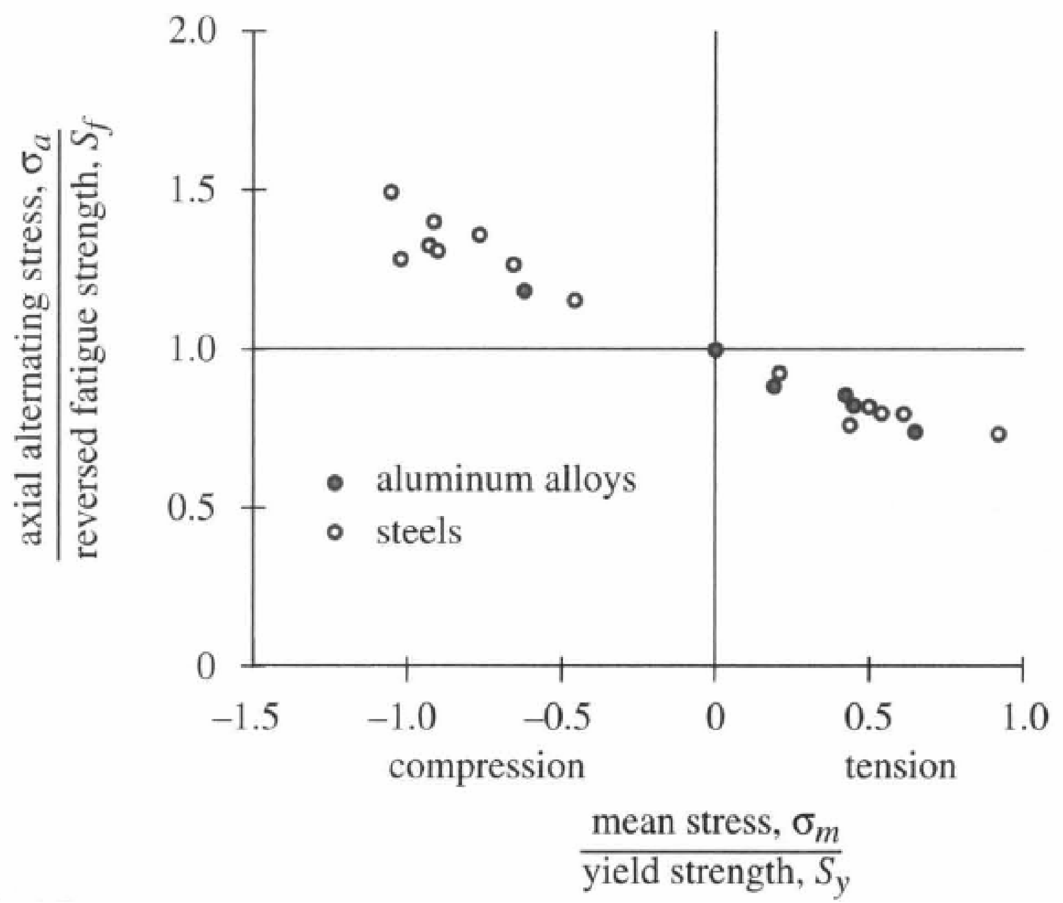
Projeto para tensões uniaxiais variadas

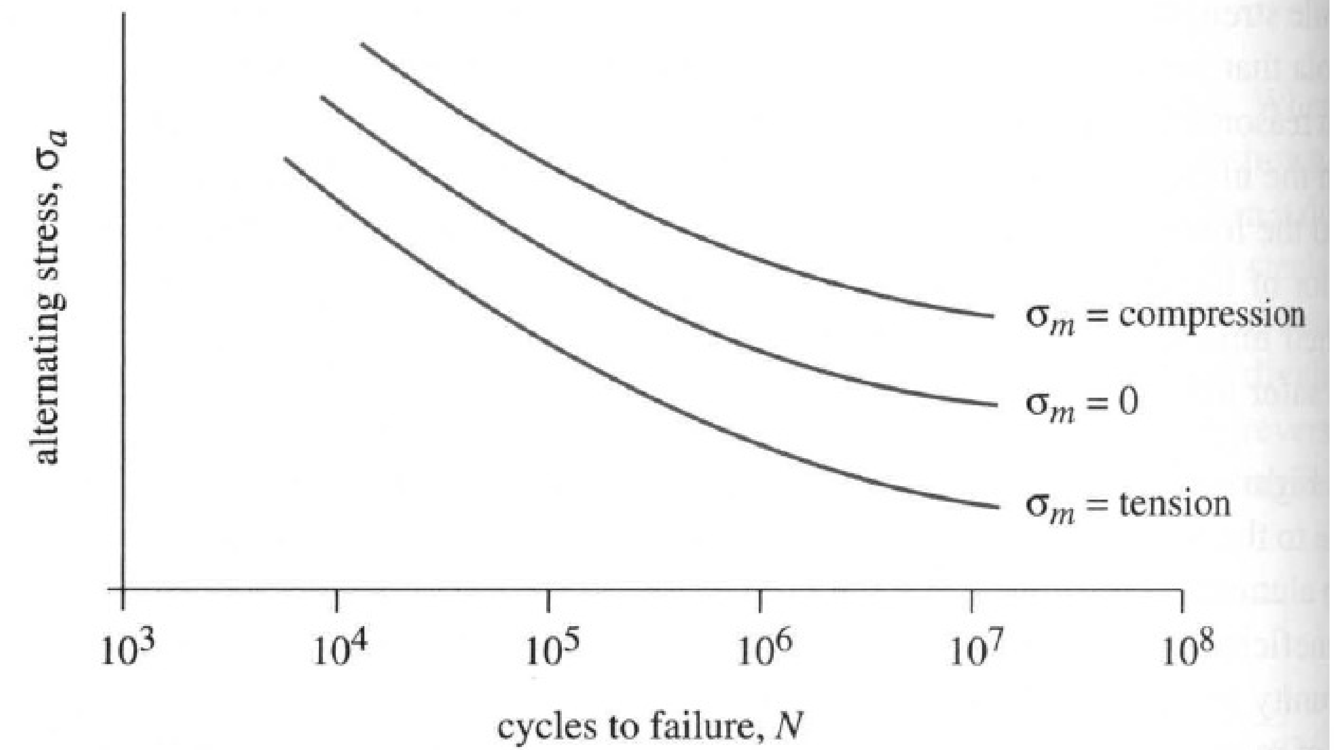
- Cargas alternadas de componente média não nula promove a variação do valor da resistência a fadiga.



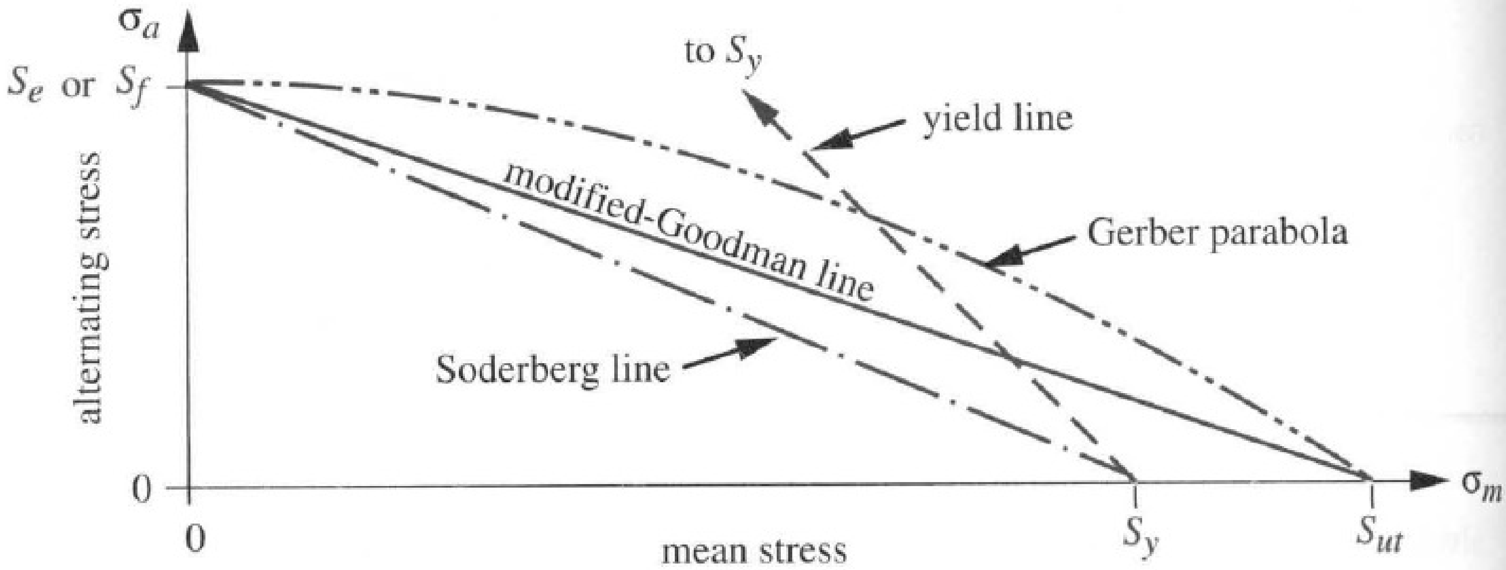
Efeito das cargas alternadas e da tensão media



Efeito das cargas alternadas e da tensão media



Curvas de fadiga no diagrama σ_m x σ_a



Gerber parabola : $$\sigma_a = S_e\left(1 - \frac{\sigma_m^2}{S_{ut}^2}\right)$$

Modified - Goodman line : $$\sigma_a = S_e\left(1 - \frac{\sigma_m}{S_{ut}}\right)$$

Soderberg line : $$\sigma_a = S_e\left(1 - \frac{\sigma_m}{S_y}\right)$$

Diagrama de Goodman modificado

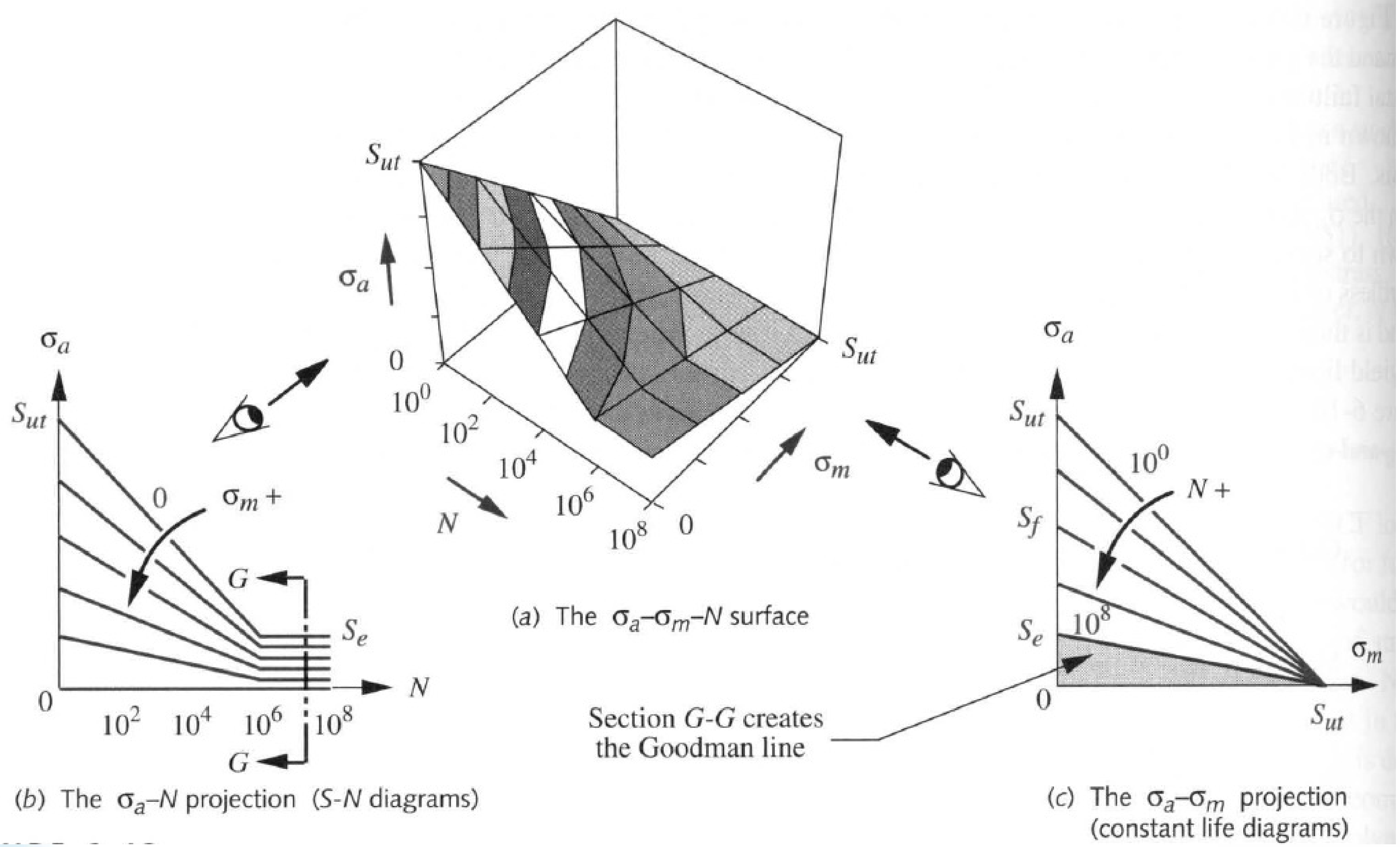
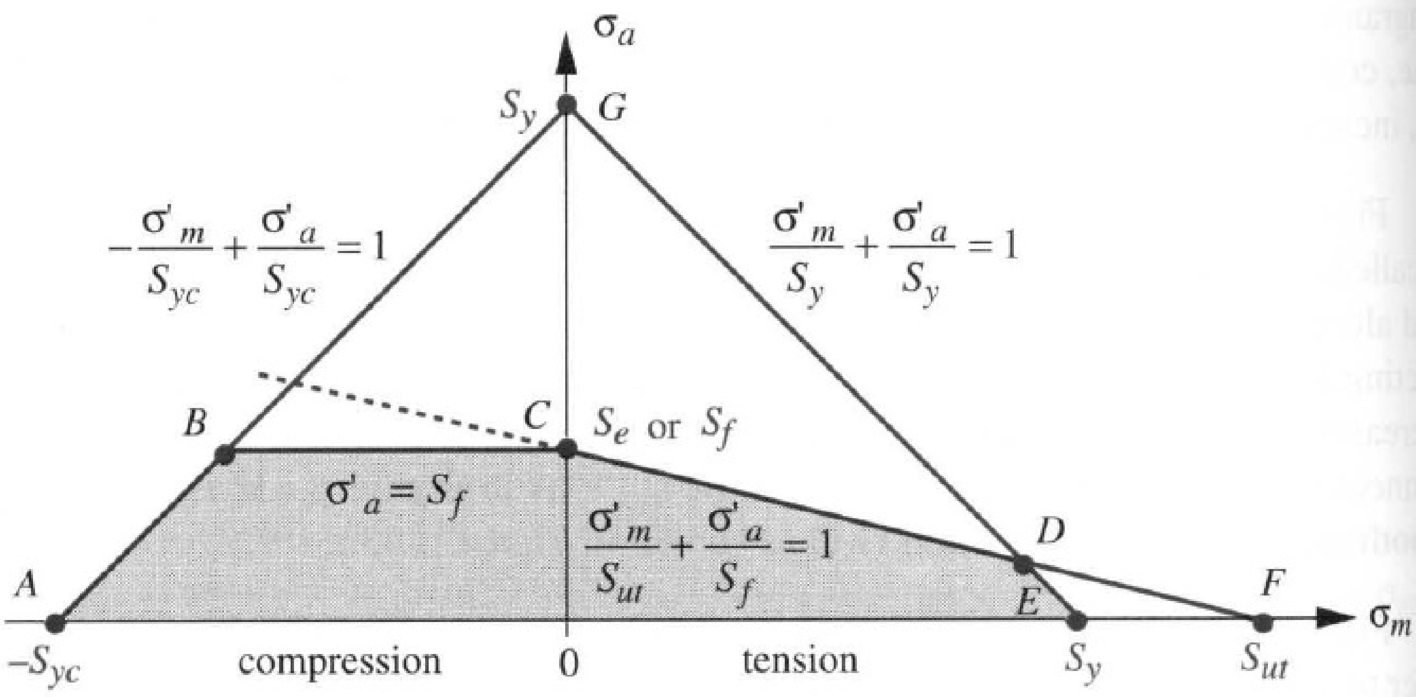
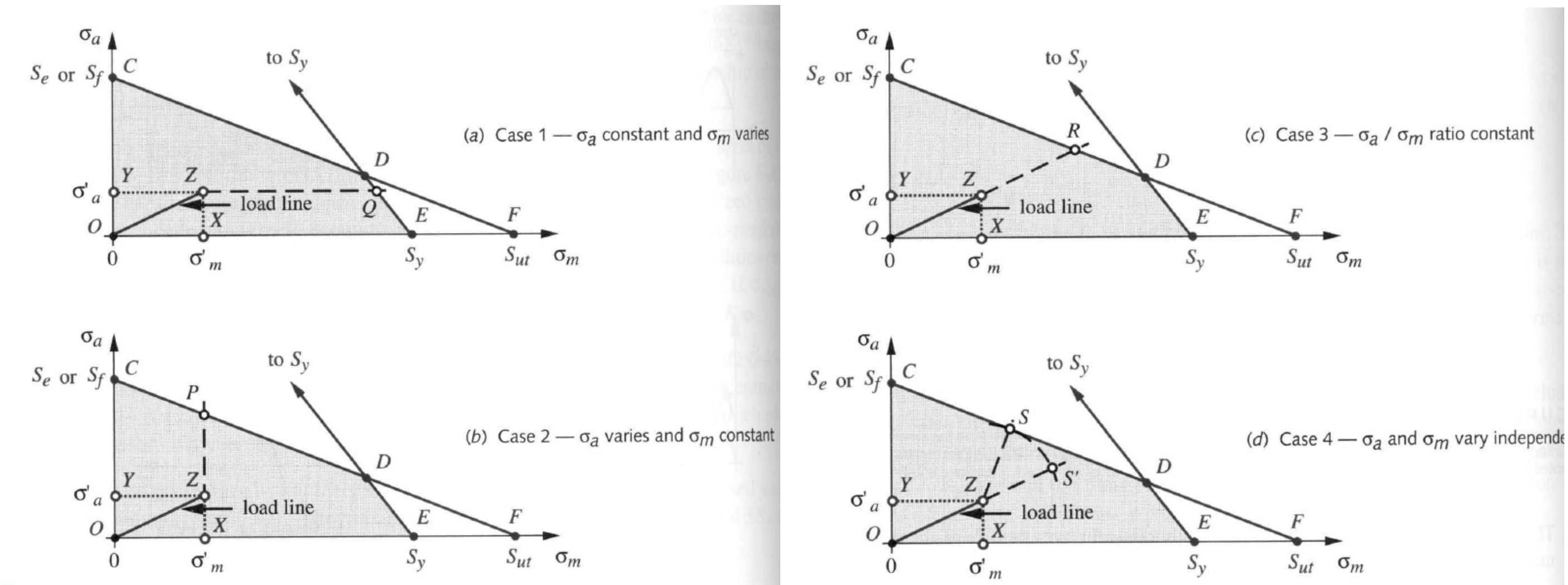


Diagrama de Goodman modificado aumentado

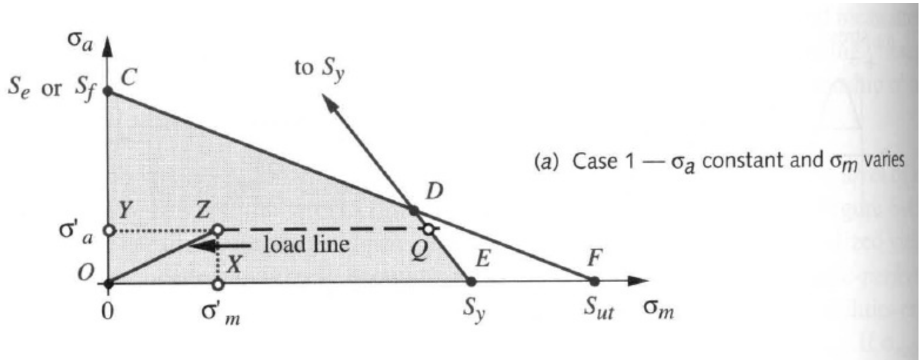


Determinação do coeficiente de segurança com tensões variadas



Caso 1

- A falha ocorre no ponto *Q*, e o coeficiente de segurança é a razao *YQ/YZ*.

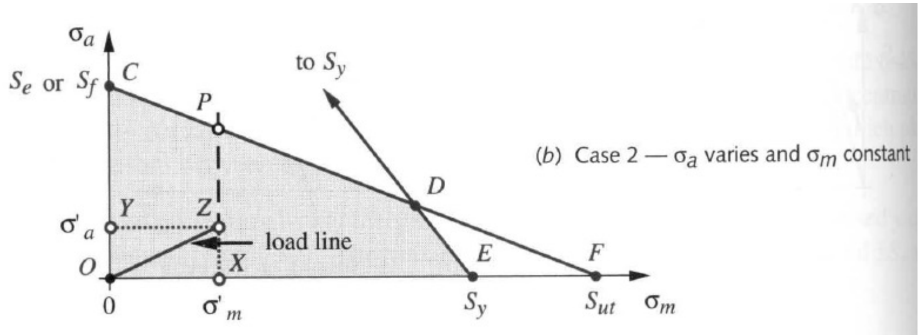


$$\sigma'_{m@Q} = \left(1 - \frac{\sigma'_a}{S_y}\right) S_y$$

$$N_f = \frac{\sigma'_{m@Q}}{\sigma'_{m@Z}} = \frac{S_y}{\sigma'_m}\left(1 - \frac{\sigma'_a}{S_y}\right)$$

Caso 2

- A falha ocorre no ponto P e o coeficiente de seguranca é a razao entre as curvas XP/XZ:



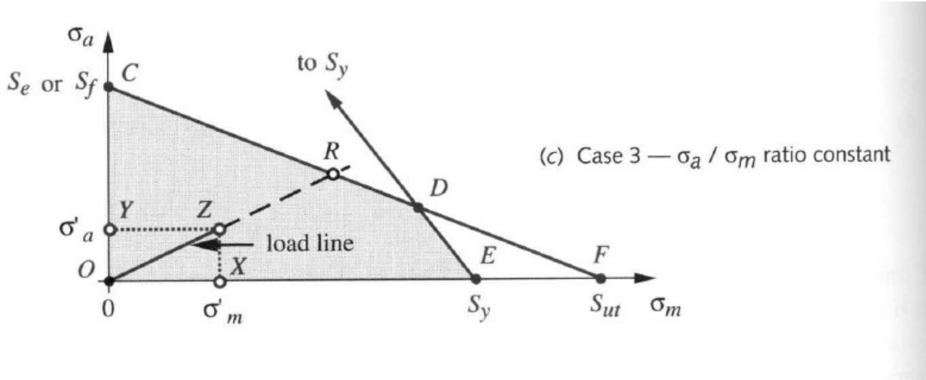
$$\sigma'_{a@P} = \left(1 - \frac{\sigma'_m}{S_{ut}}\right) S_f$$

$$N_f = \frac{\sigma'_{a@P}}{\sigma'_{a@Z}} = \frac{S_f}{\sigma'_a}\left(1 - \frac{\sigma'_m}{S_{ut}}\right)$$

Caso 3

- A falha ocorre no ponto R e o coeficiente de seguranca é a razao OR/OY.

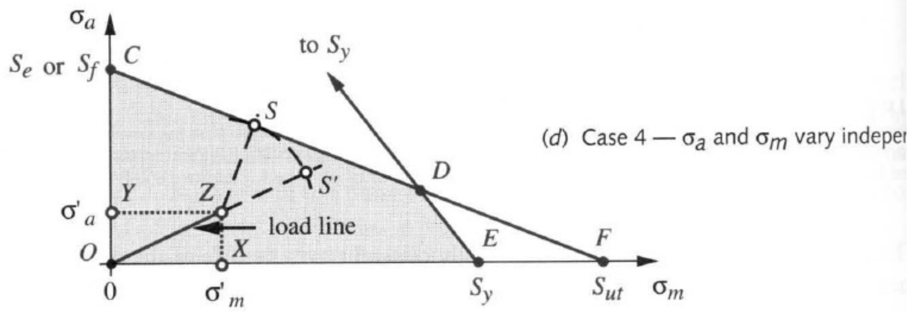
$$\sigma'_{m@R} = \frac{S_f}{\dfrac{\sigma'_a}{\sigma'_m} + \dfrac{S_f}{S_{ut}}}$$



$$N_f = \frac{\sigma'_{m@R}}{\sigma'_{m@Z}} = \frac{S_f S_{ut}}{\sigma'_a S_{ut} + \sigma'_m S_f}$$

Caso 4

- Como tanto a tensao media quanto a alternada varia devemos tomar o ponto mais próximo S.



$$\sigma'_{m@S} = \frac{S_{ut}\left(S_f^{\,2} - S_f\sigma'_a + S_{ut}\sigma'_m\right)}{S_f^{\,2} + S_{ut}^{\,2}}$$

$$\sigma'_{a@S} = -\frac{S_f}{S_{ut}}\left(\sigma'_{m@S}\right) + S_f$$

$$ZS = \sqrt{\left(\sigma'_m - \sigma'_{m@S}\right)^2 + \left(\sigma'_a - \sigma'_{a@S}\right)^2}$$

$$OZ = \sqrt{\left(\sigma'_a\right)^2 + \left(\sigma'_m\right)^2}$$

$$N_f = \frac{OZ + ZS}{OZ}$$

Tensoes alternadas multiaxiais simples

- Experimentos comprovam que a teoria da energia de distorcao máxima é satisfatoria para este caso logo:

$$\sigma'_a = \sqrt{\sigma_{1_a}^2 + \sigma_{2_a}^2 + \sigma_{3_a}^2 - \sigma_{1_a}\sigma_{2_a} - \sigma_{2_a}\sigma_{3_a} - \sigma_{1_a}\sigma_{3_a}}$$

and for the two-dimensional case:

$$\sigma'_a = \sqrt{\sigma_{1_a}^2 - \sigma_{1_a}\sigma_{2_a} + \sigma_{2_a}^2}$$

$$N_f = \frac{S_n}{\sigma'_a}$$

Tensoes variadas multiaxiais simples

- Metodo de Sines:
  - Calcula-se tensoes medias e alternadas equivalentes que sao aplicadas em um diagrama de Goodman modificado:

$$\sigma'_a = \sqrt{\frac{\left(\sigma_{x_a} - \sigma_{y_a}\right)^2 + \left(\sigma_{y_a} - \sigma_{z_a}\right)^2 + \left(\sigma_{z_a} - \sigma_{x_a}\right)^2 + 6\left(\tau_{xy_a}^2 + \tau_{yz_a}^2 + \tau_{zx_a}^2\right)}{2}}$$

$$\sigma'_m = \sigma_{x_m} + \sigma_{y_m} + \sigma_{z_m}$$

and, for a biaxial stress state:

$$\sigma'_a = \sqrt{\sigma_{x_a}^2 + \sigma_{y_a}^2 - \sigma_{x_a}\sigma_{y_a} + 3\tau_{xy_a}^2}$$

$$\sigma'_m = \sigma_{x_m} + \sigma_{y_m}$$

Tensoes variadas multiaxiais simples

- Metodo de Von Mises:

$$\sigma'_a = \sqrt{\frac{\left(\sigma_{x_a} - \sigma_{y_a}\right)^2 + \left(\sigma_{y_a} - \sigma_{z_a}\right)^2 + \left(\sigma_{z_a} - \sigma_{x_a}\right)^2 + 6\left(\tau_{xy_a}^2 + \tau_{yz_a}^2 + \tau_{zx_a}^2\right)}{2}}$$

$$\sigma'_m = \sqrt{\frac{\left(\sigma_{x_m} - \sigma_{y_m}\right)^2 + \left(\sigma_{y_m} - \sigma_{z_m}\right)^2 + \left(\sigma_{z_m} - \sigma_{x_m}\right)^2 + 6\left(\tau_{xy_m}^2 + \tau_{yz_m}^2 + \tau_{zx_m}^2\right)}{2}}$$

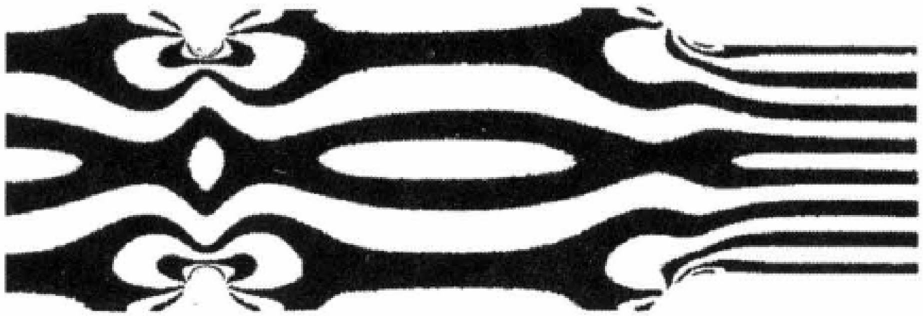
or for a biaxial stress state using:

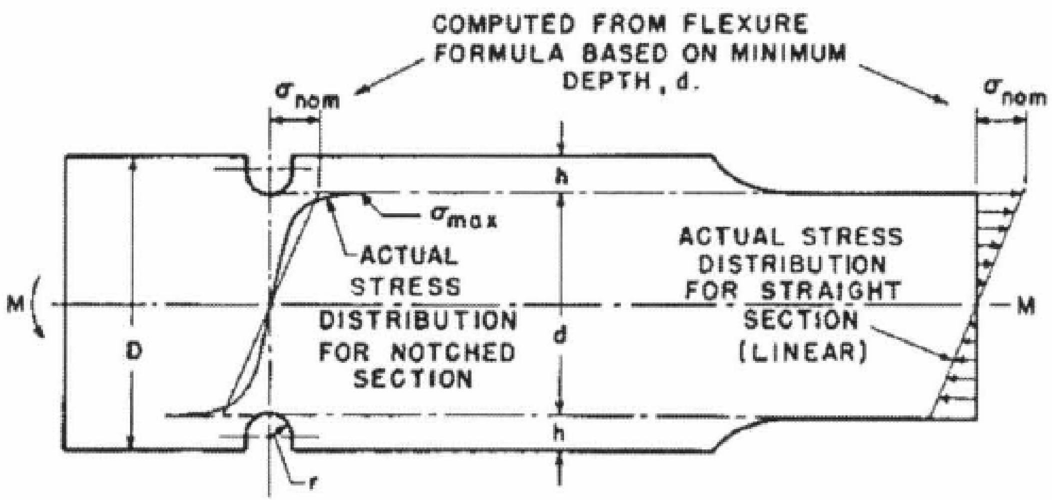
$$\sigma'_a = \sqrt{\sigma_{x_a}^2 + \sigma_{y_a}^2 - \sigma_{x_a}\sigma_{y_a} + 3\tau_{xy_a}^2}$$

$$\sigma'_m = \sqrt{\sigma_{x_m}^2 + \sigma_{y_m}^2 - \sigma_{x_m}\sigma_{y_m} + 3\tau_{xy_m}^2}$$

Concentradores de Tensões

Linhas de distribuição de carga





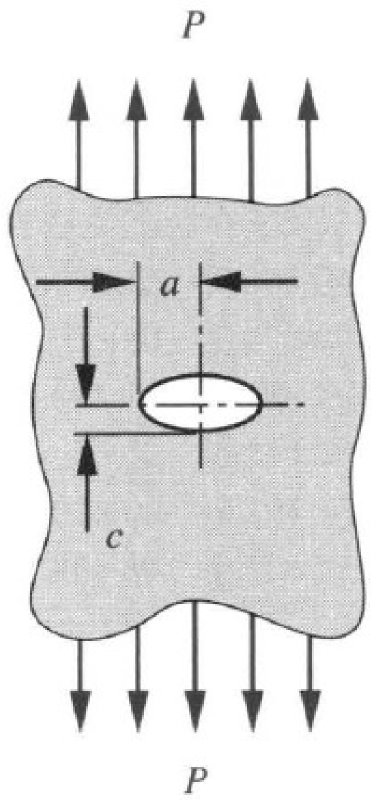
$$\sigma_{max} = K_t\,\sigma_{nom}$$

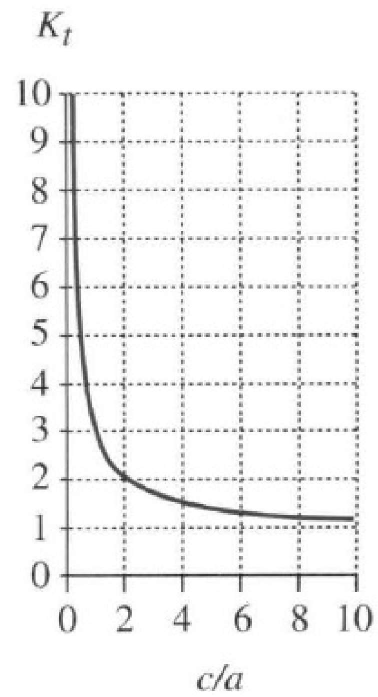
$$\tau_{max} = K_{ts}\,\tau_{nom}$$

Concentração de tensões na solicitação estática

- Materiais dúcteis:
  - Escoam localmente em torno do ponto de aumento de tensão.
  - Os efeitos da concentração de tensões são minimizados.
- Materiais frágeis:
  - Não escoam localmente;
  - Os efeitos da concentração de tensão são importantes mesmo sob carga estática;
  - Para materiais fundidos os efeitos dos concentradores de tensão podem ser ignorados (presença de defeitos pré-exixtentes)

Determinação dos fatores geométricos de concentração de tensões

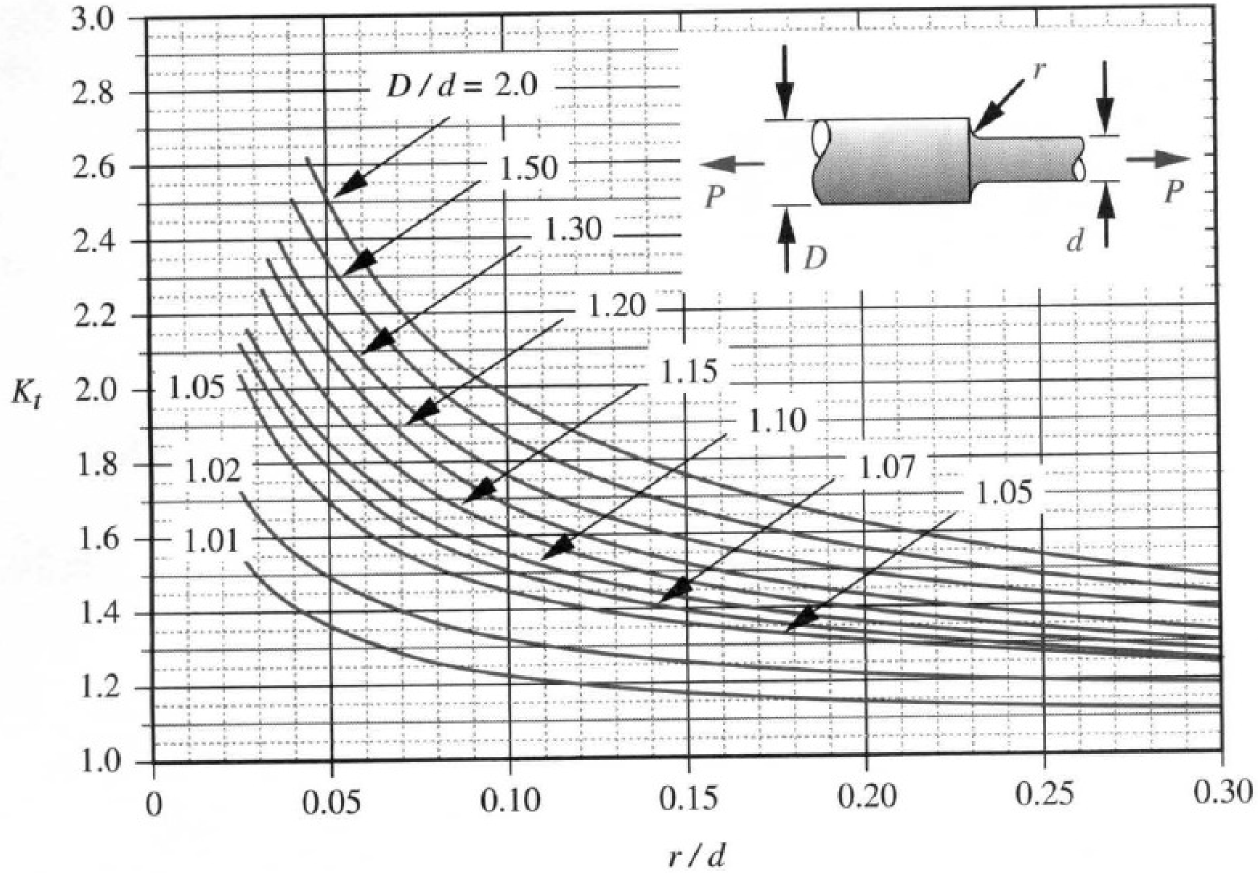




$$K_t = 1 + 2\left(\frac{a}{c}\right)$$

Teoria da Elasticidade

Dados experimentais para concentradores de tensões

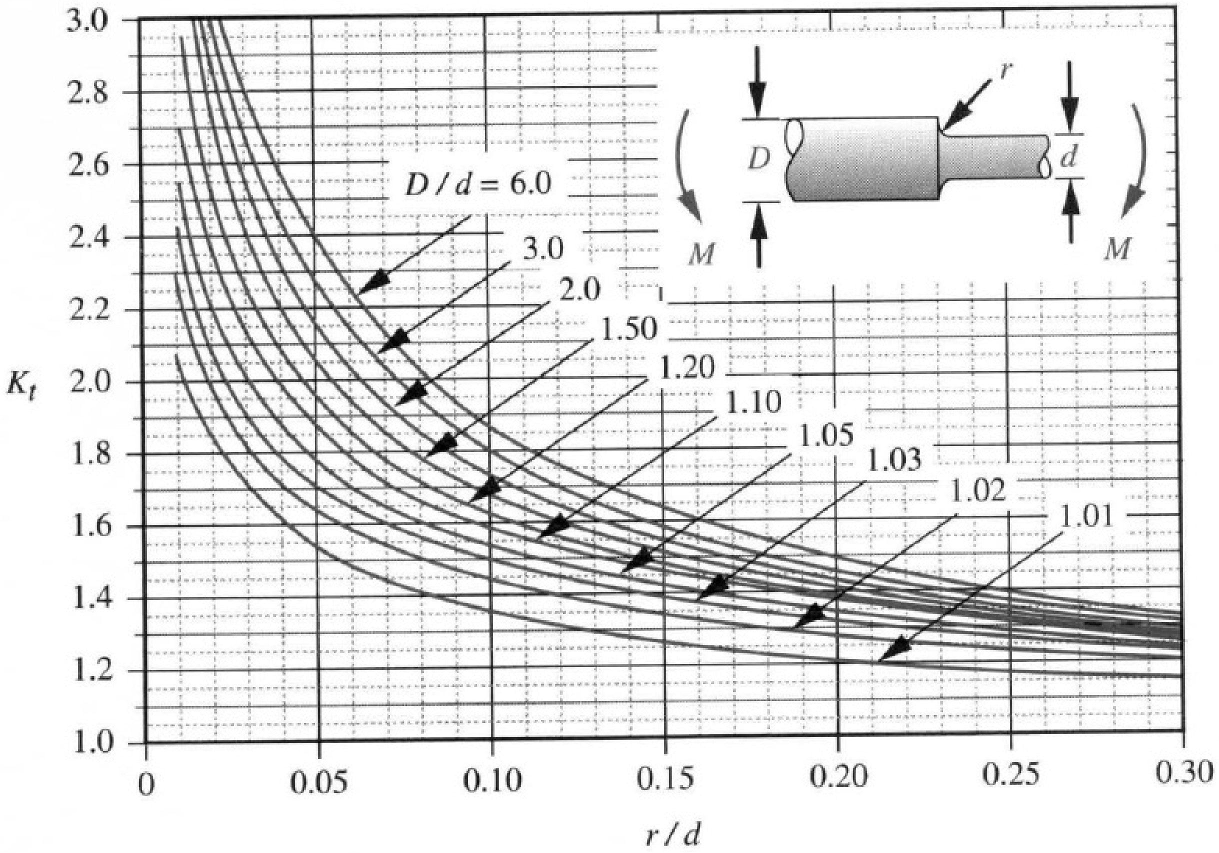


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
2.00	1.014 70	–0.300 35
1.50	0.999 57	–0.282 21
1.30	0.996 82	–0.257 51
1.20	0.962 72	–0.255 27
1.15	0.980 84	–0.224 85
1.10	0.984 50	–0.208 18
1.07	0.984 98	–0.195 48
1.05	1.004 80	–0.170 76
1.02	1.012 20	–0.124 74
1.01	0.984 13	–0.104 74

Dados experimentais para concentradores de tensões

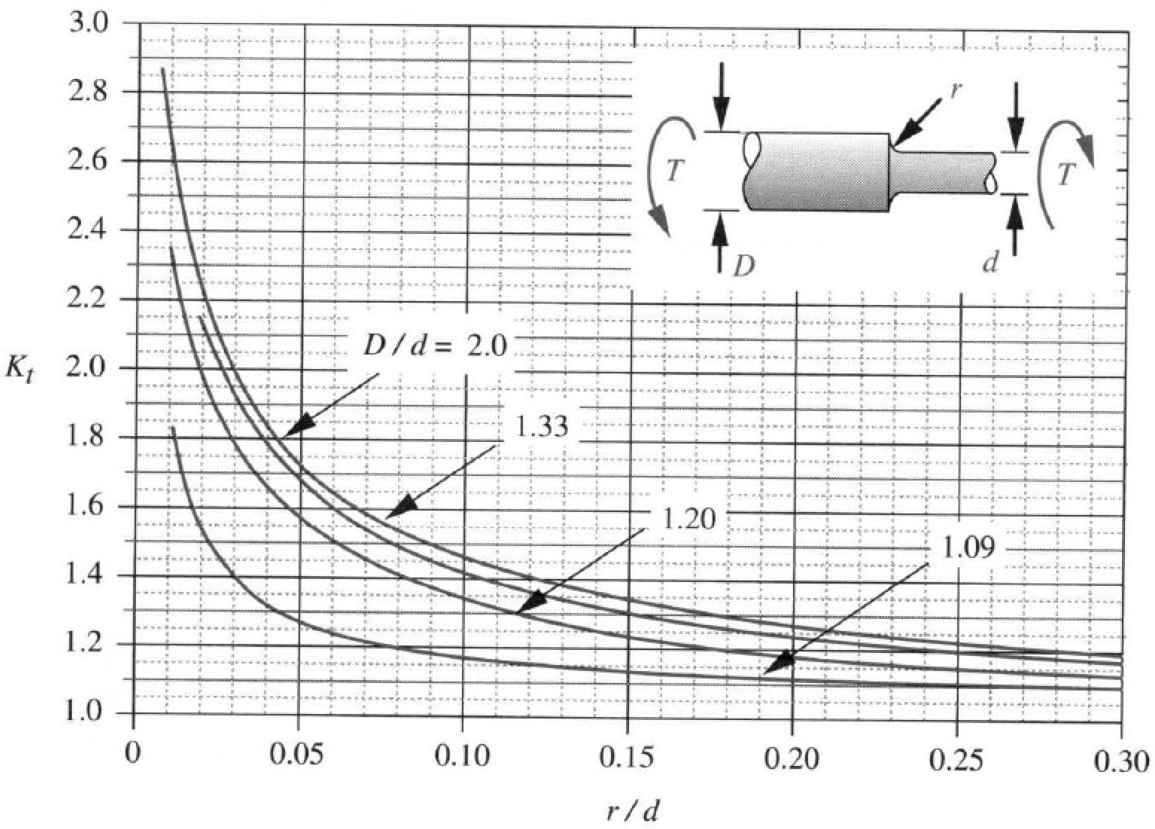


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
6.00	0.878 68	−0.332 43
3.00	0.893 34	−0.308 60
2.00	0.908 79	−0.285 98
1.50	0.938 36	−0.257 59
1.20	0.970 98	−0.217 96
1.10	0.951 20	−0.237 57
1.07	0.975 27	−0.209 58
1.05	0.981 37	−0.196 53
1.03	0.980 61	−0.183 81
1.02	0.960 48	−0.177 11
1.01	0.919 38	−0.170 32

Dados experimentais para concentradores de tensões

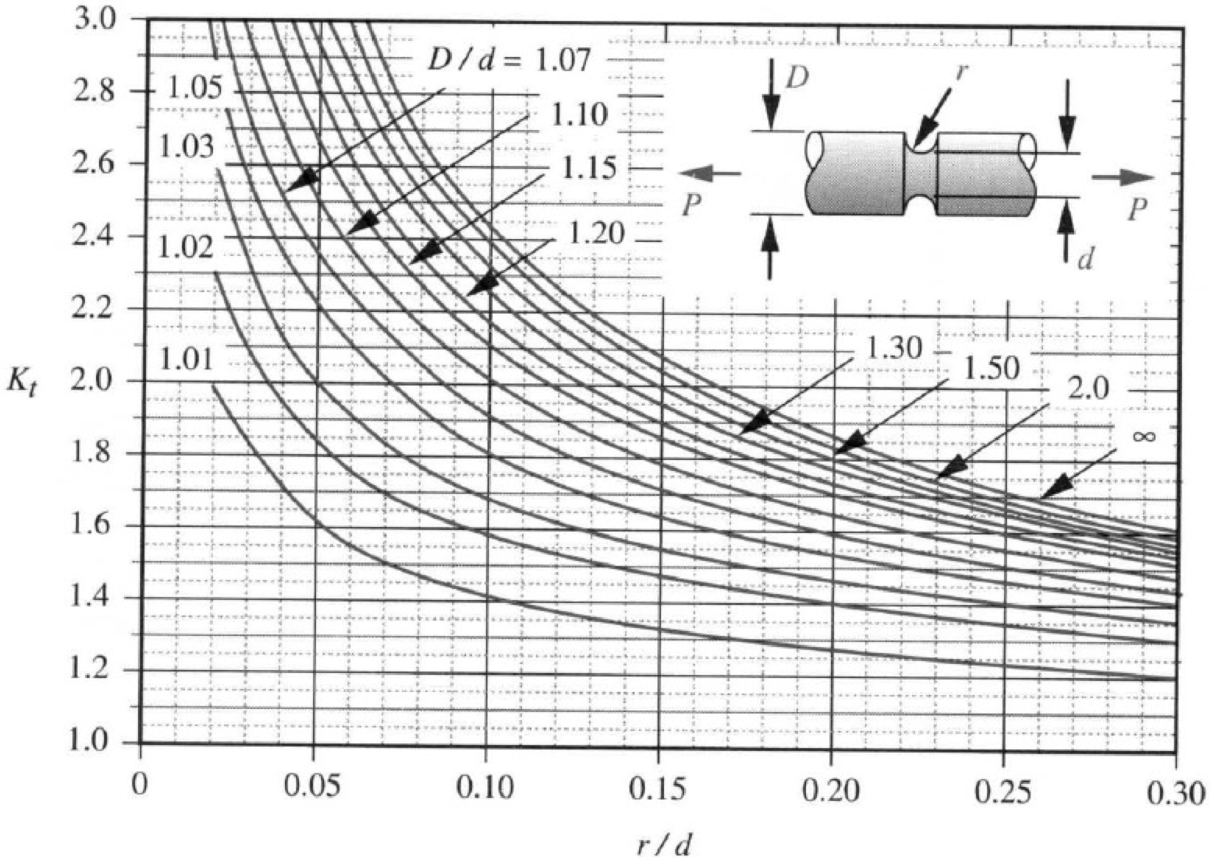


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
2.00	0.863 31	–0.238 65
1.33	0.848 97	–0.231 61
1.20	0.834 25	–0.216 49
1.09	0.903 37	–0.126 92

Dados experimentais para concentradores de tensões

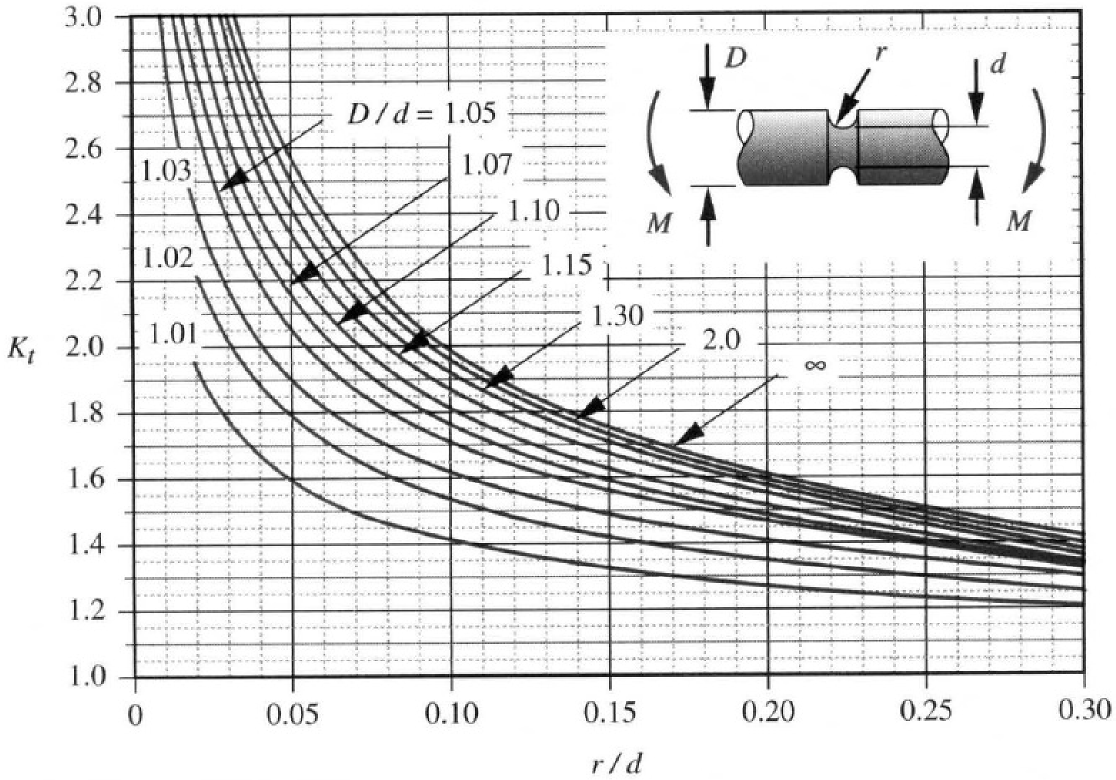


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
∞	0.993 72	−0.393 52
2.00	0.993 83	−0.382 31
1.50	0.998 08	−0.369 55
1.30	1.004 90	−0.355 45
1.20	1.010 70	−0.337 65
1.15	1.026 30	−0.316 73
1.10	1.027 20	−0.294 84
1.07	1.023 80	−0.276 18
1.05	1.027 20	−0.252 56
1.03	1.036 70	−0.216 03
1.02	1.037 90	−0.187 55
1.01	1.000 30	−0.156 09

Dados experimentais para concentradores de tensões

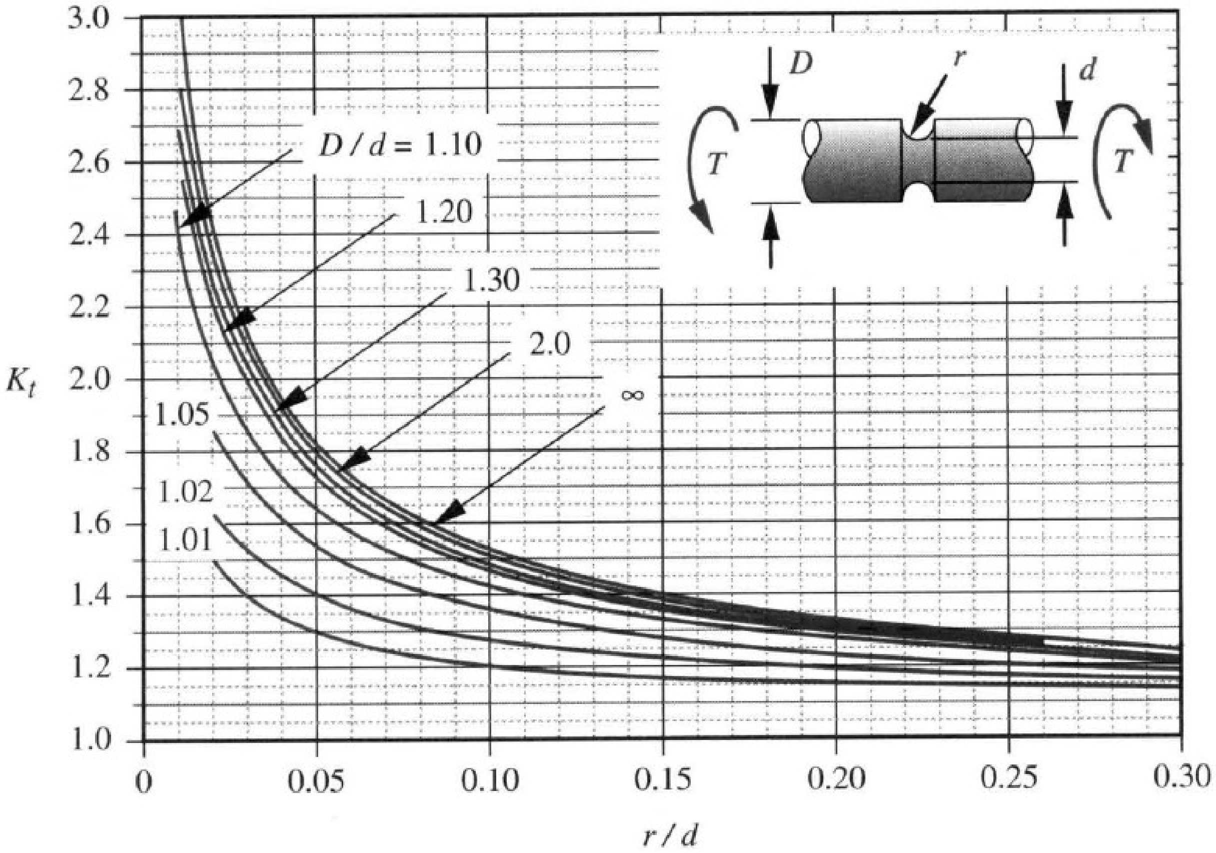


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
∞	0.948 01	−0.333 02
2.00	0.936 19	−0.330 66
1.50	0.938 94	−0.323 80
1.30	0.942 99	−0.315 04
1.20	0.946 81	−0.305 82
1.15	0.953 11	−0.297 39
1.12	0.955 73	−0.288 86
1.10	0.954 54	−0.282 68
1.07	0.967 74	−0.264 52
1.05	0.987 55	−0.241 34
1.03	0.990 33	−0.215 17
1.02	0.977 53	−0.197 93
1.01	0.993 93	−0.152 38

Dados experimentais para concentradores de tensões

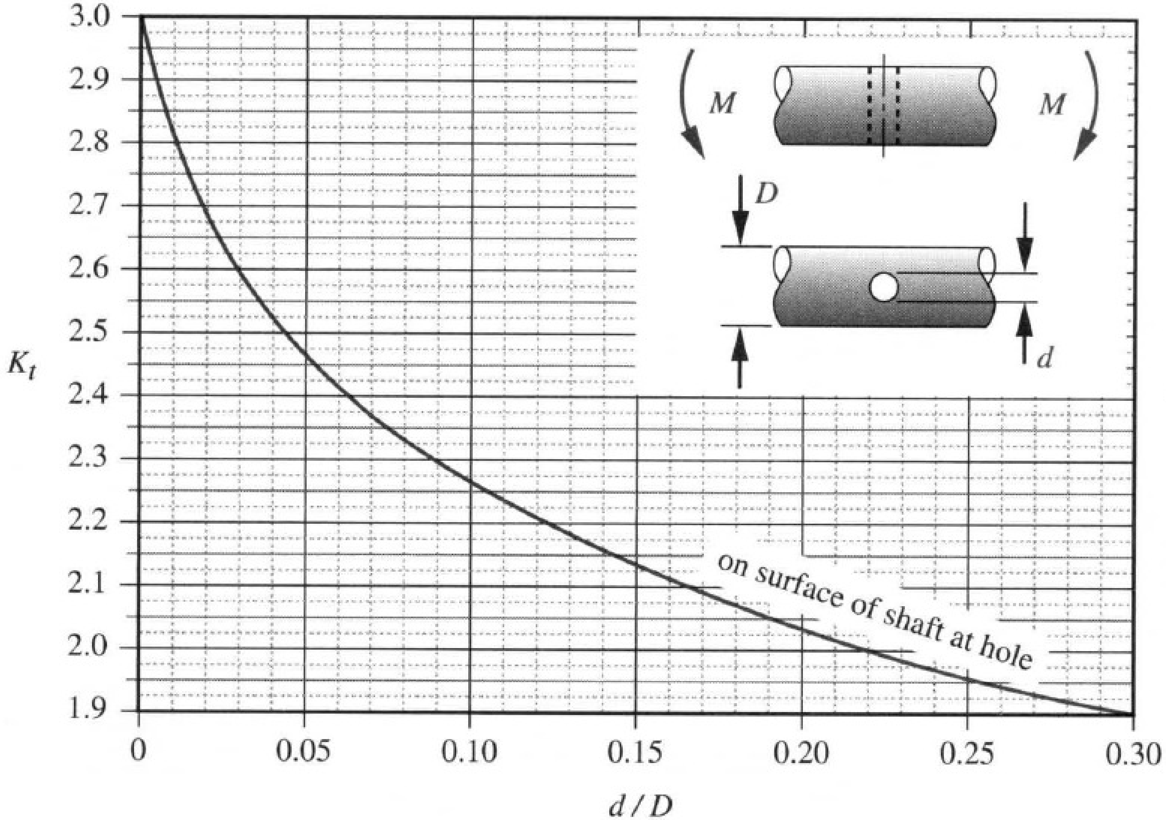


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

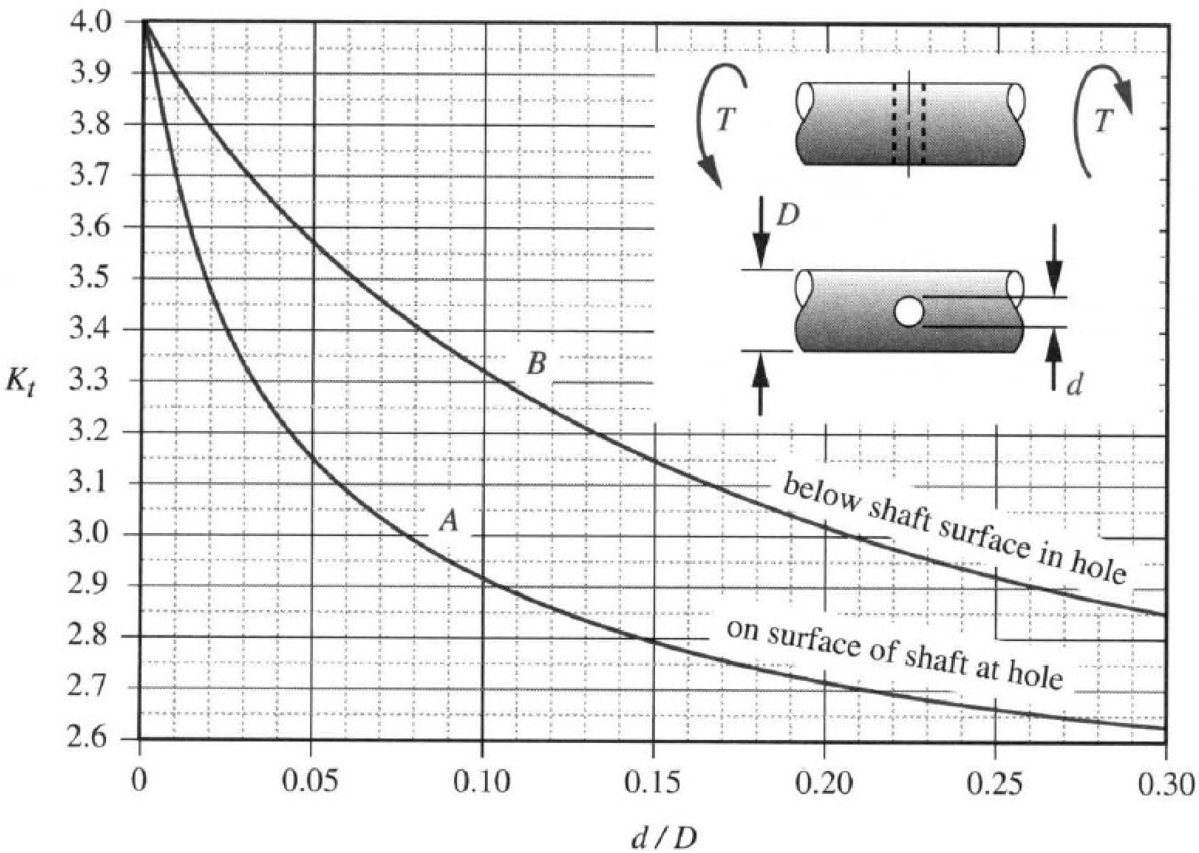
D/d	A	b
∞	0.881 26	–0.252 04
2.00	0.890 35	–0.240 75
1.30	0.894 60	–0.232 67
1.20	0.901 82	–0.223 34
1.10	0.923 11	–0.197 40
1.05	0.938 53	–0.169 41
1.02	0.968 77	–0.126 05
1.01	0.972 45	–0.101 62

Dados experimentais para concentradores de tensões



$$K_t \cong 1.589\,90 - 0.635\,50 \log\left(\frac{d}{D}\right)$$

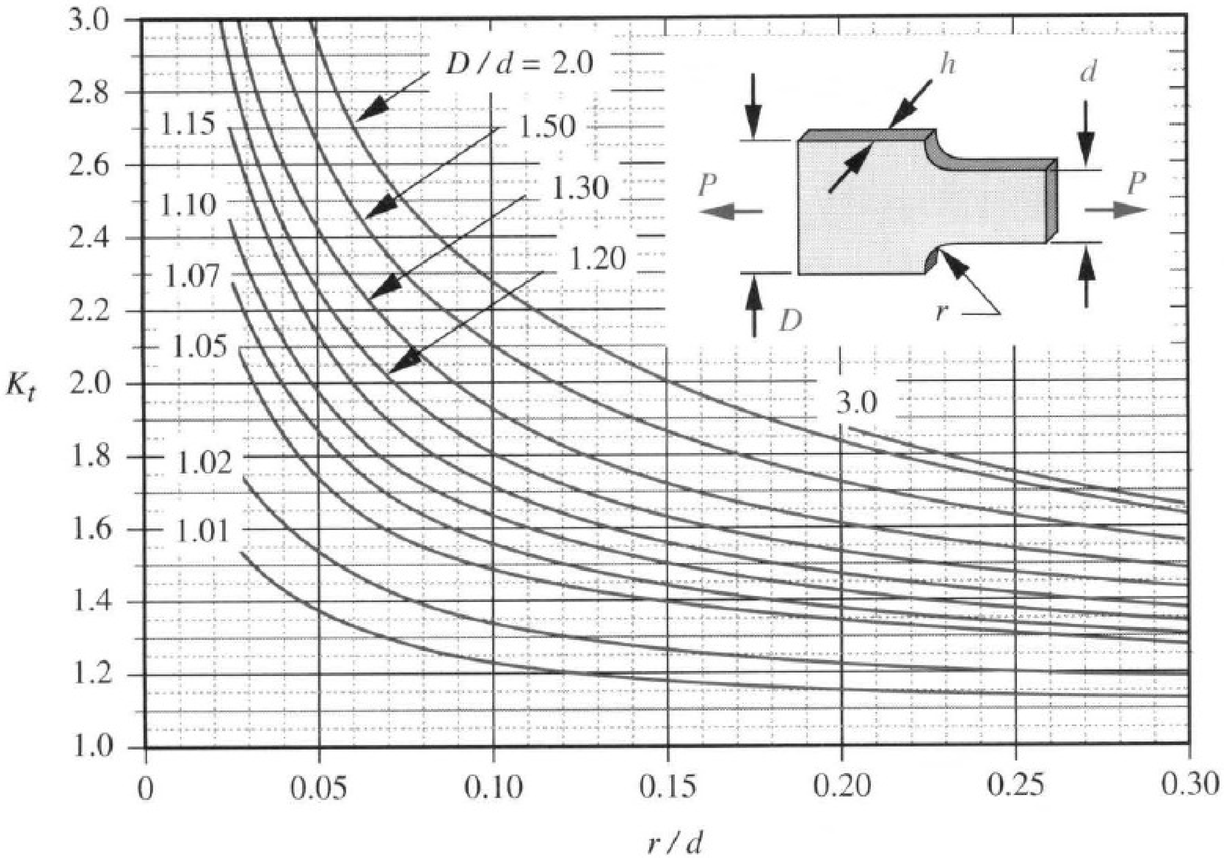
Dados experimentais para concentradores de tensões



$$K_{t_B} \cong 3.9702 - 9.292\frac{d}{D} + 27.159\left(\frac{d}{D}\right)^2 + 30.231\left(\frac{d}{D}\right)^3 - 393.19\left(\frac{d}{D}\right)^4 + 650.39\left(\frac{d}{D}\right)^5 + 15.451\left(\frac{d}{D}\right)^6$$

$$K_{t_A} \cong 3.921\,50 - 24.435\frac{d}{D} + 234.06\left(\frac{d}{D}\right)^2 - 1\,200.5\left(\frac{d}{D}\right)^3 + 3\,059.5\left(\frac{d}{D}\right)^4 - 3042.4\left(\frac{d}{D}\right)^5$$

Dados experimentais para concentradores de tensões

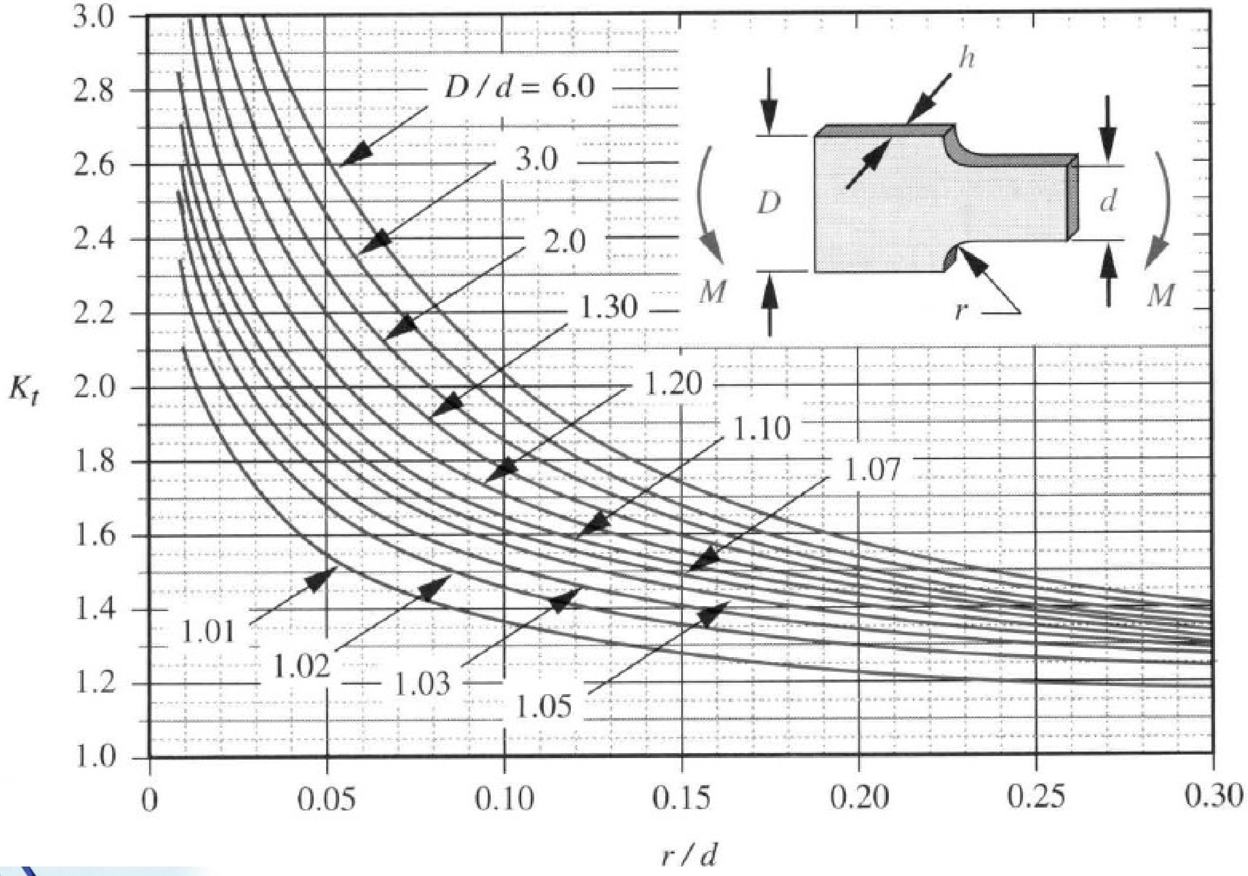


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
2.00	1.099 60	−0.320 77
1.50	1.076 90	−0.295 58
1.30	1.054 40	−0.270 21
1.20	1.035 10	−0.250 84
1.15	1.014 20	−0.239 35
1.10	1.013 00	−0.215 35
1.07	1.014 50	−0.193 66
1.05	0.987 97	−0.138 48
1.02	1.025 90	−0.169 78
1.01	0.976 62	−0.106 56

Dados experimentais para concentradores de tensões

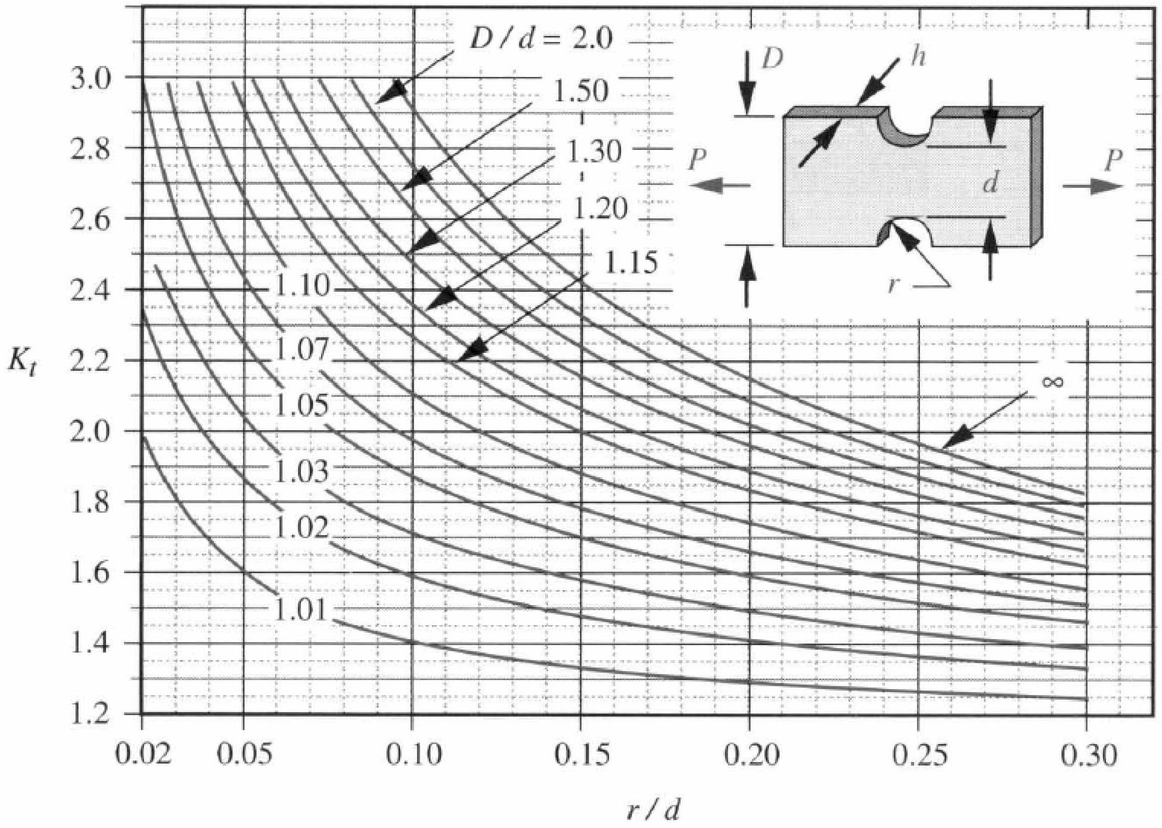


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
6.00	0.895 79	−0.358 47
3.00	0.907 20	−0.333 33
2.00	0.932 32	−0.303 04
1.30	0.958 80	−0.272 69
1.20	0.995 90	−0.238 29
1.10	1.016 50	−0.215 48
1.07	1.019 90	−0.203 33
1.05	1.022 60	−0.191 56
1.03	1.016 60	−0.178 02
1.02	0.995 28	−0.170 13
1.01	0.966 89	−0.154 17

Dados experimentais para concentradores de tensões

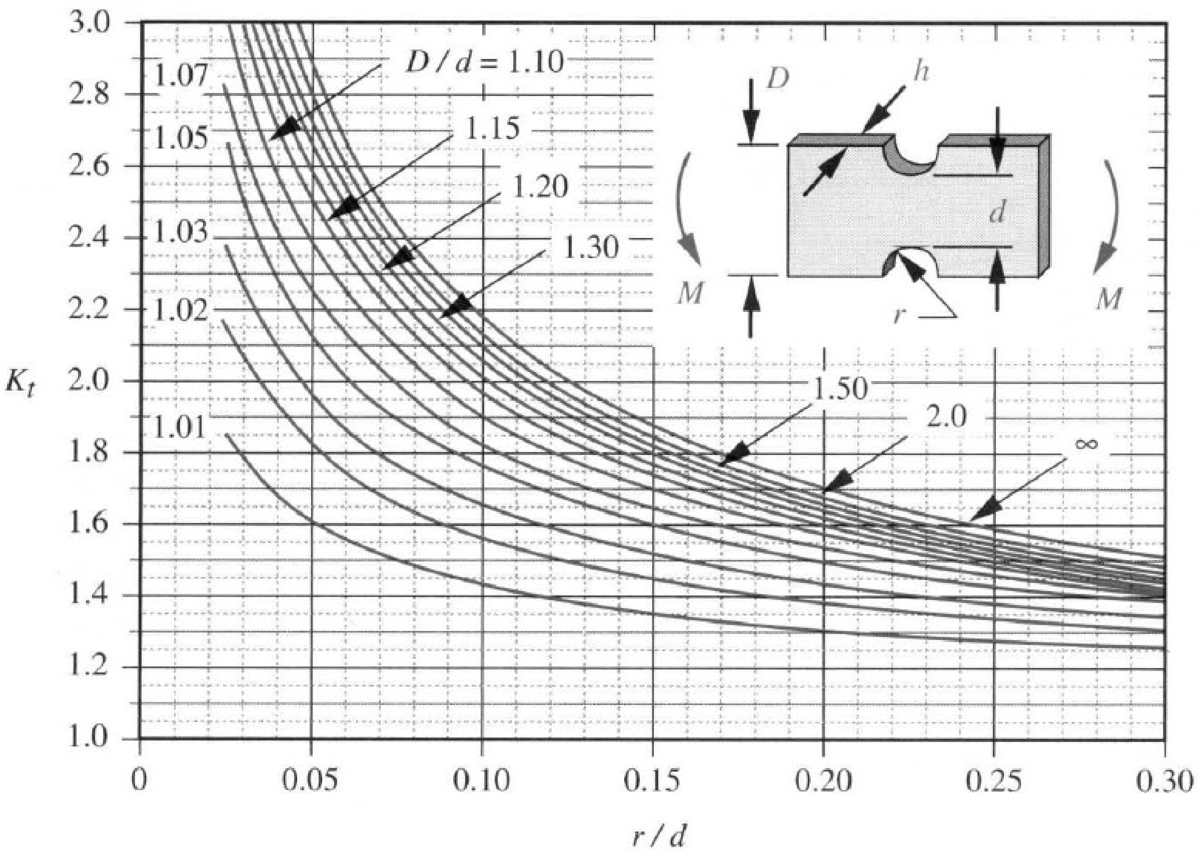


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
∞	1.109 50	−0.417 12
3.00	1.113 90	−0.409 23
2.00	1.133 90	−0.385 86
1.50	1.132 60	−0.365 92
1.30	1.158 60	−0.332 60
1.20	1.147 50	−0.315 07
1.15	1.095 20	−0.325 17
1.10	1.085 10	−0.299 97
1.07	1.091 20	−0.268 57
1.05	1.090 60	−0.241 63
1.03	1.051 80	−0.222 16
1.02	1.054 00	−0.188 79
1.01	1.042 60	−0.141 45

Dados experimentais para concentradores de tensões

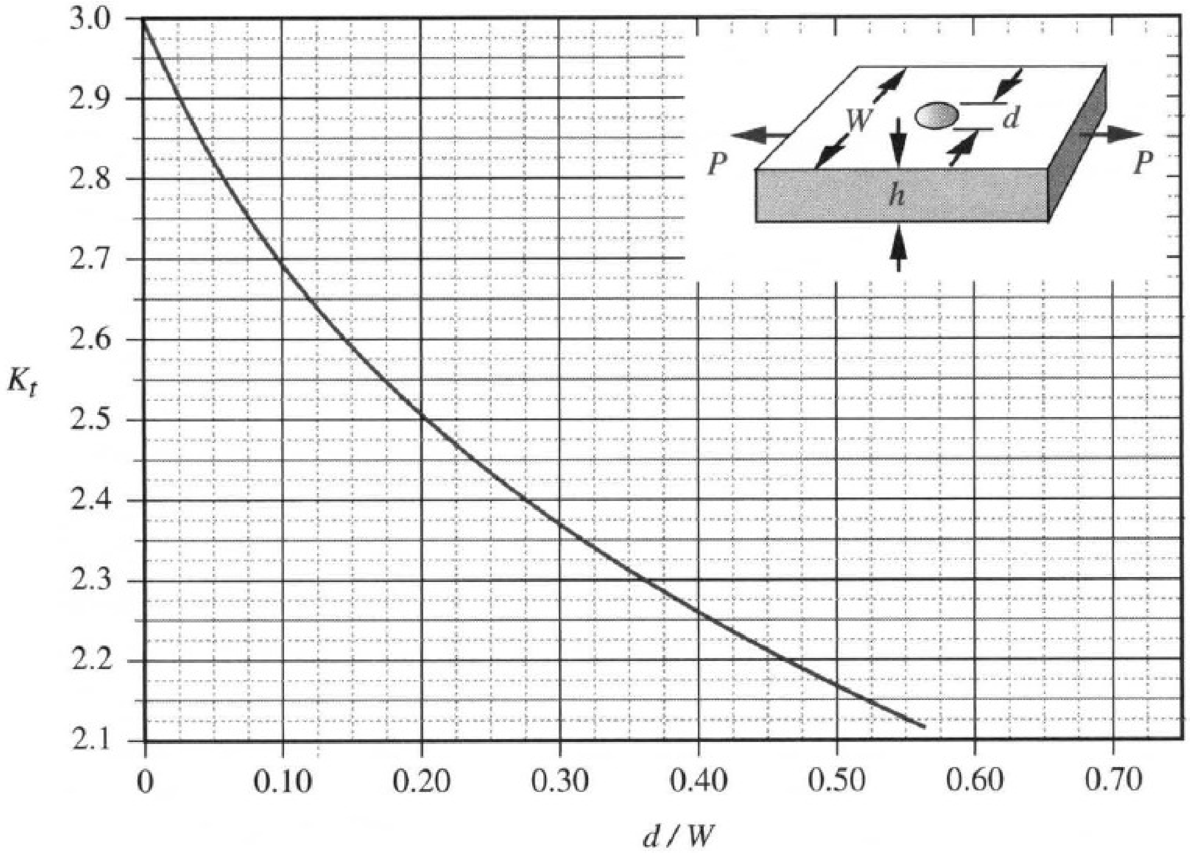


$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
∞	0.970 79	–0.356 72
3.00	0.971 94	–0.350 47
2.00	0.968 01	–0.349 15
1.50	0.983 15	–0.333 95
1.30	0.982 88	–0.326 06
1.20	0.990 55	–0.313 19
1.15	0.993 04	–0.302 63
1.10	1.007 10	–0.283 79
1.07	1.014 70	–0.261 45
1.05	1.025 00	–0.240 08
1.03	1.029 40	–0.211 61
1.02	1.037 40	–0.184 28
1.01	1.060 50	–0.133 69

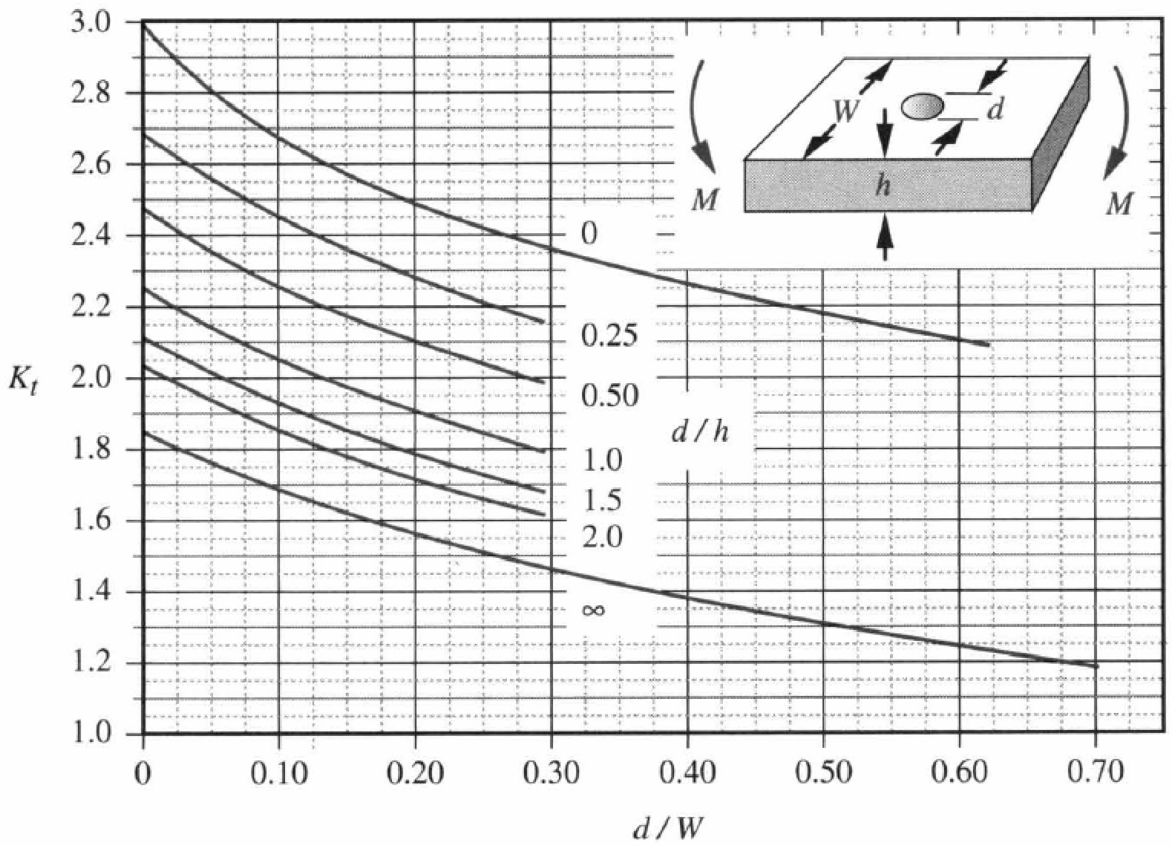
Dados experimentais para concentradores de tensões



for $\dfrac{d}{W} \leq 0.65$:

$$K_t \cong 3.003\,9 - 3.753\frac{d}{W} + 7.973\,5\left(\frac{d}{W}\right)^2 - 9.265\,9\left(\frac{d}{W}\right)^3 + 1.814\,5\left(\frac{d}{W}\right)^4 + 2.968\,4\left(\frac{d}{W}\right)^5$$

Dados experimentais para concentradores de tensões



for $\dfrac{d}{h} \Rightarrow 0$ and $\dfrac{d}{W} \leq 0.65$:

$$K_t \cong 2.994\,7 - 3.483\,3\frac{d}{W} + 5.826\,8\left(\frac{d}{W}\right)^2 - 4.198\,6\left(\frac{d}{W}\right)^3$$

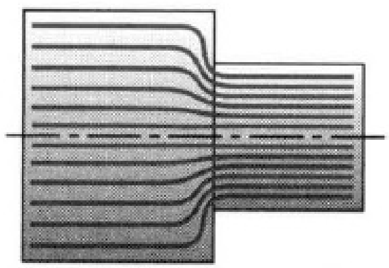
for $\dfrac{d}{h} \geq 0.25$: $K_t \cong A e^{[b(d/W)]}$

where:

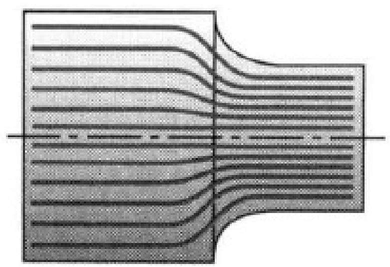
d/h	A	b
0.25	2.687 50	–0.751 28
0.50	2.466 20	–0.772 15
1.00	2.240 00	–0.787 39
1.50	2.024 30	–0.808 21
2.00	2.105 60	–0.798 78
∞	1.808 20	–0.667 02

Regras para evitar a concentração de tensões

- Evitar variações abruptas na seção transversal
- Evitar cantos agudos
- Utilizar raios de adoçamento

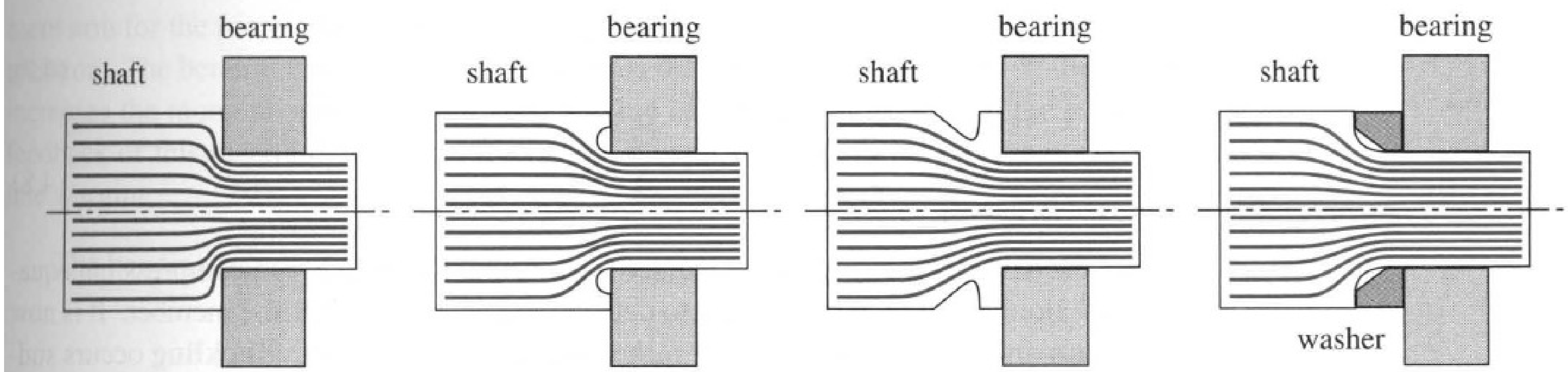


(a) Force flow around a sharp corner



(b) Force flow around a radiused corner

Regras para evitar a concentração de tensões



(a) Stress concentration at a sharp corner

(b) Stress concentration reduced with radius

(c) Stress concentration reduced with groove

(d) Stress concentration reduced with washer

Concentração de tensões para cargas dinâmicas

- Definição da sensibilidade ao entalhe:

$$q = \frac{K_f - 1}{K_t - 1}$$

$$K_f = 1 + q(K_t - 1)$$

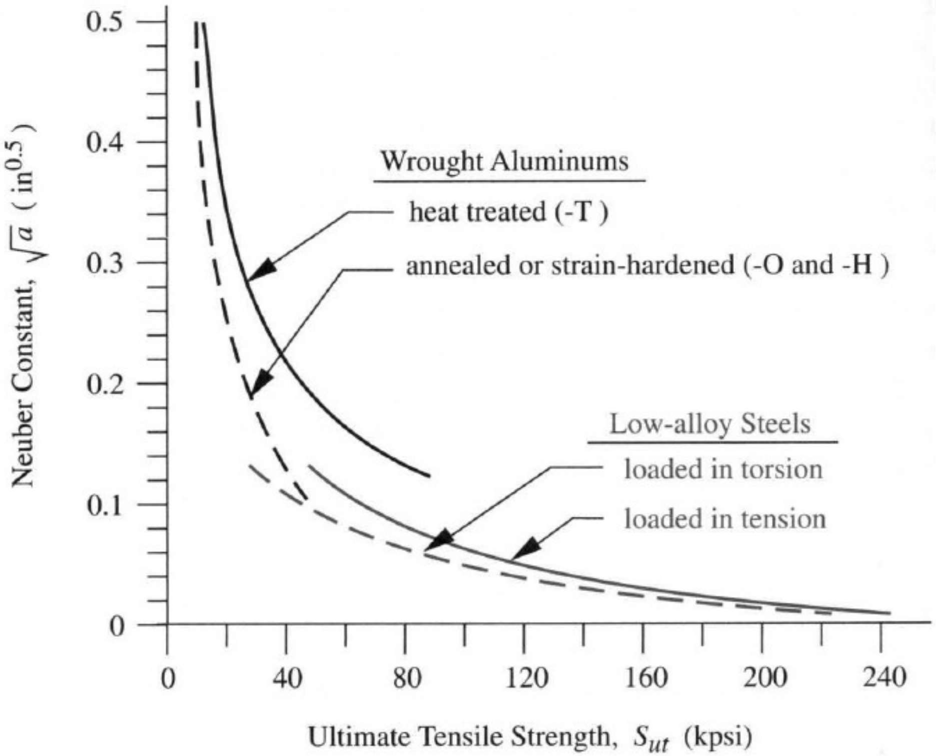
- Logo a resistência do material será

$$\sigma = K_f \sigma_{nom}$$

$$\tau = K_{fs} \tau_{nom}$$

Fator de sensibilidade ao entalhe

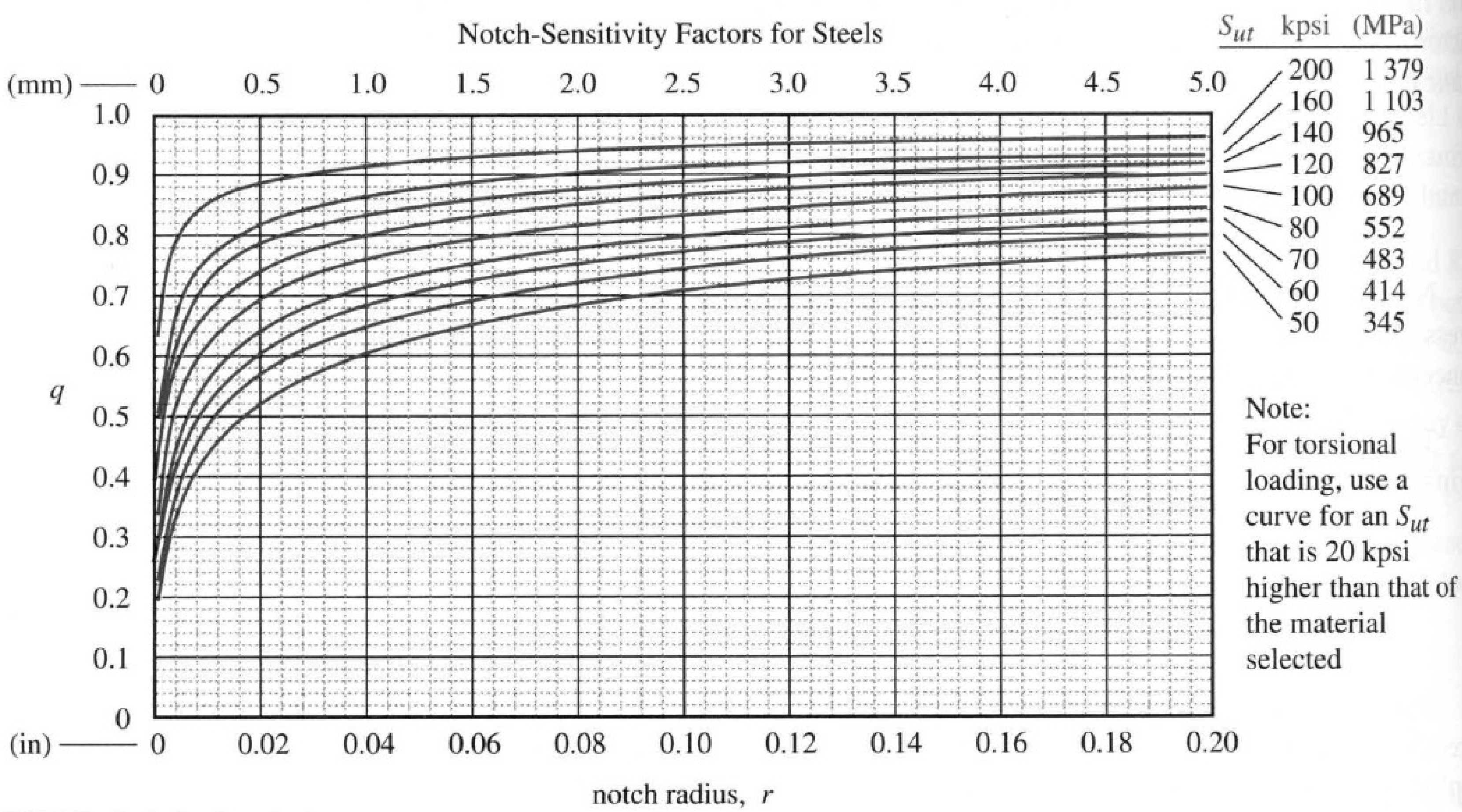
$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$



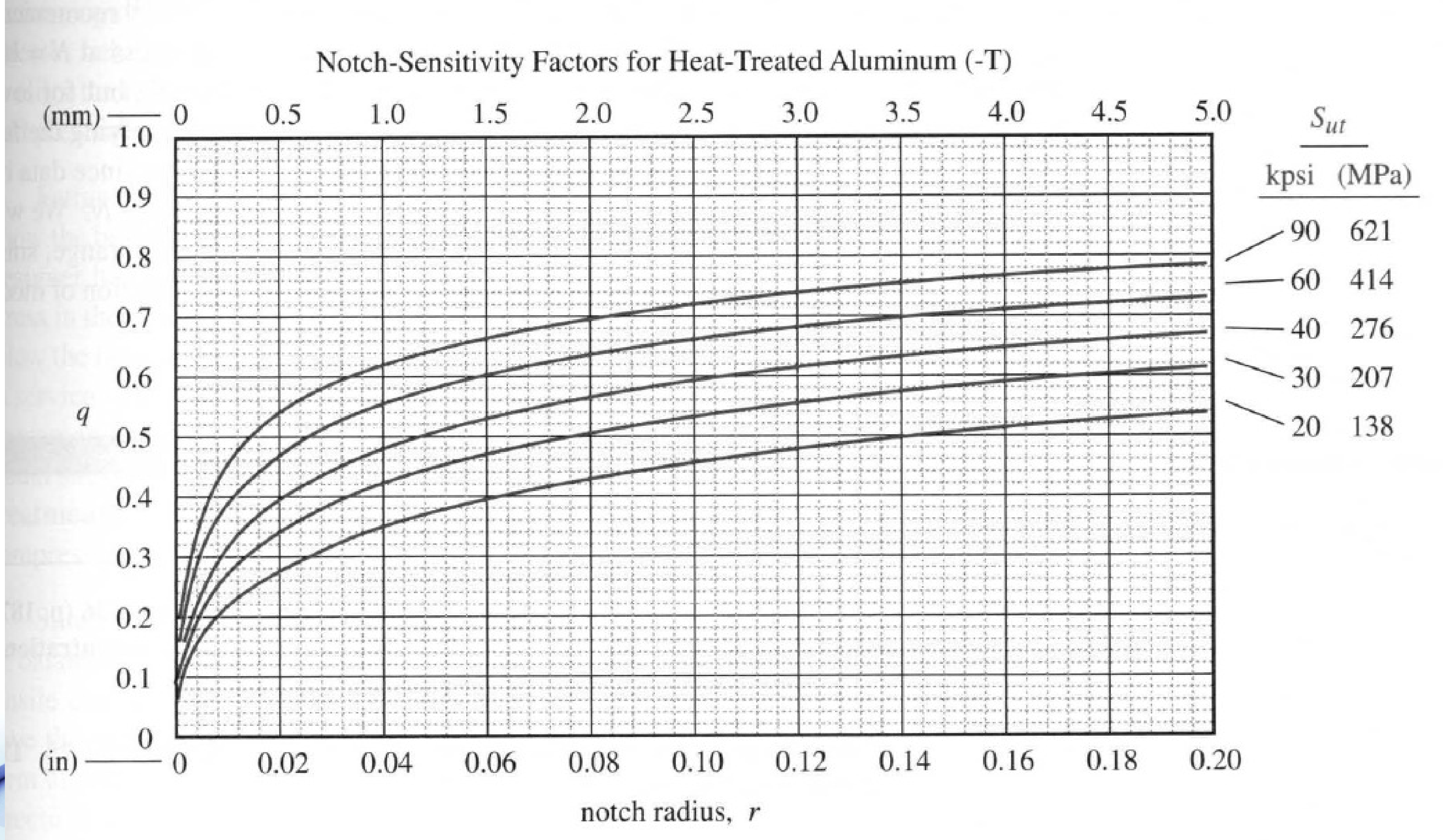
Neuber's Constant
for Steels

S_{ut} (ksi)	$\sqrt{a}$ ($in^{0.5}$)
50	0.130
55	0.118
60	0.108
70	0.093
80	0.080
90	0.070
100	0.062
110	0.055
120	0.049
130	0.044
140	0.039
160	0.031
180	0.024
200	0.018
220	0.013
240	0.009

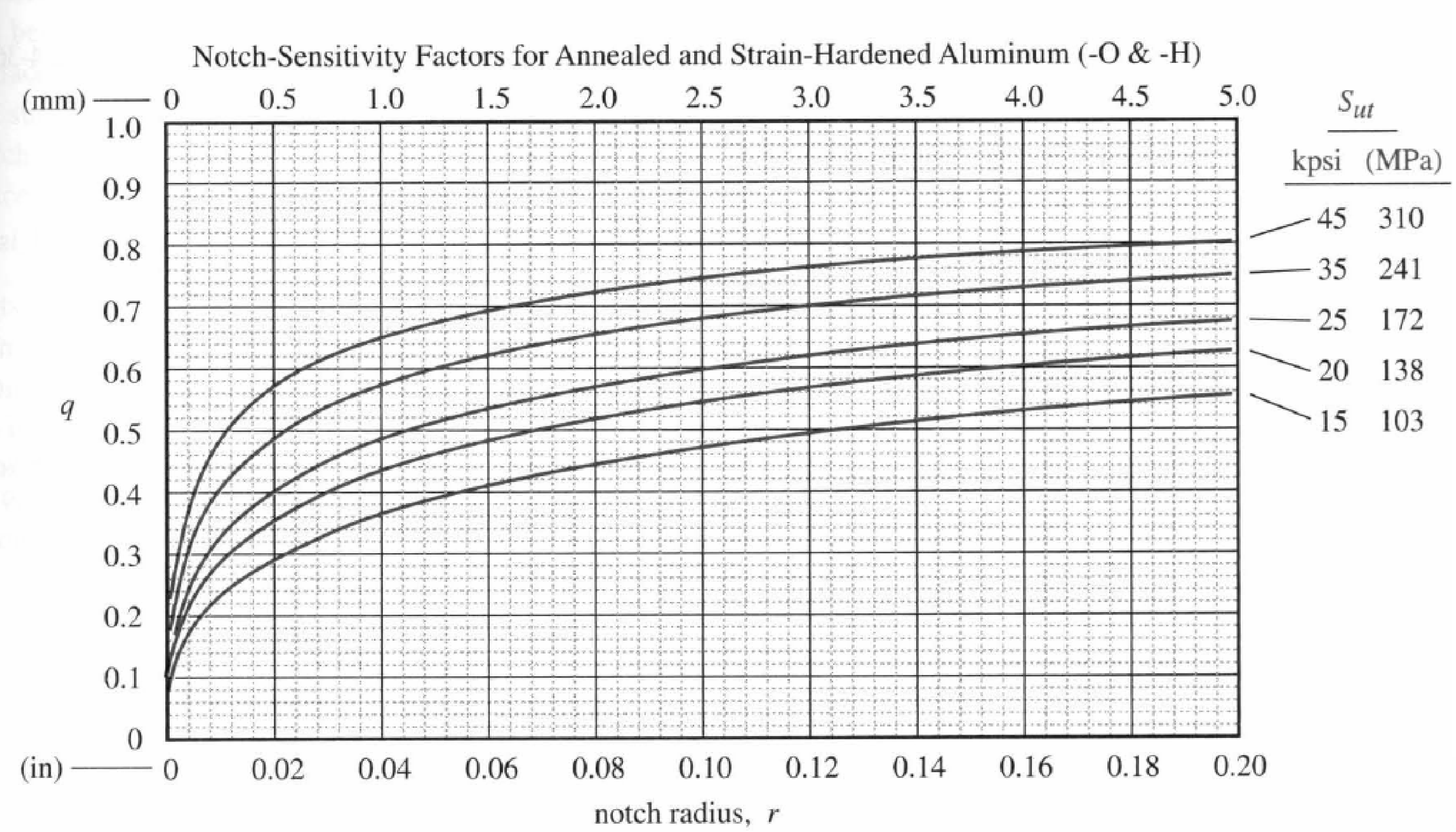
Fator de sensibilidade ao entalhe



Fator de sensibilidade ao entalhe



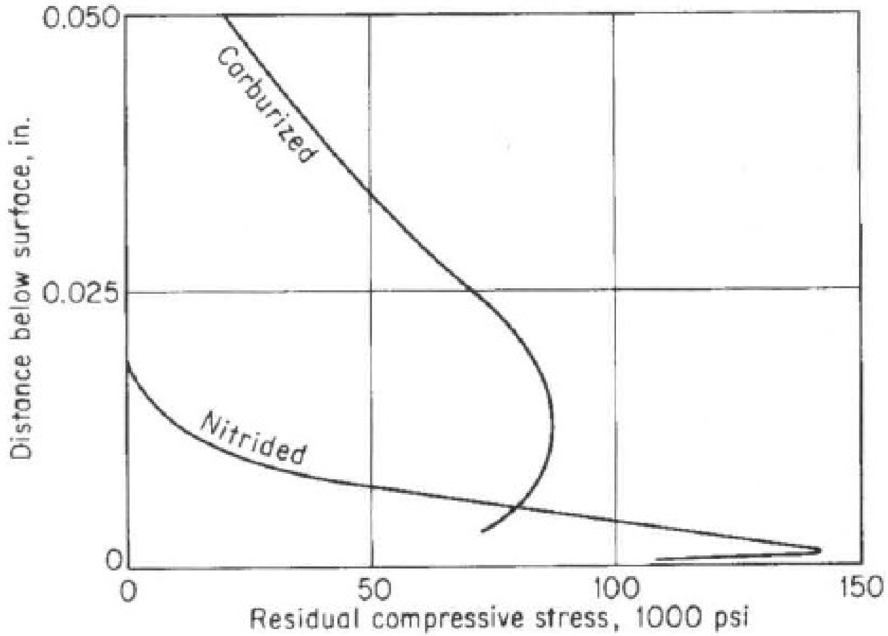
Fator de sensibilidade ao entalhe



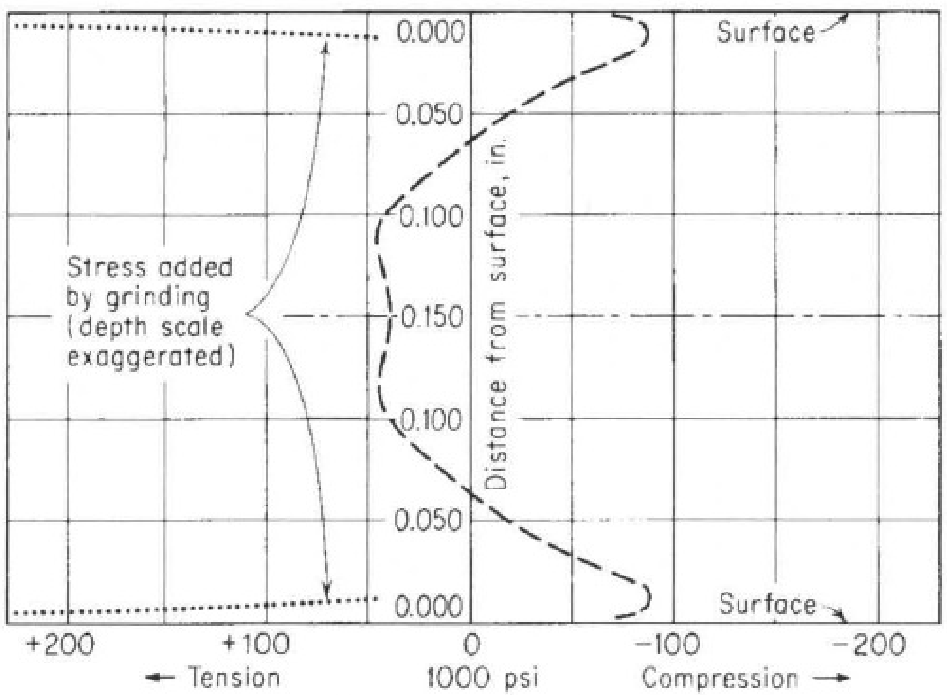
Tensões Residuais

- Uso de tratamentos térmicos e superficiais para gerar tensões residuais de compressão;
- Têmpera Completa;
- Têmpera Superficial;
- Tratamentos superficiais

Têmpera Superficial



Acabamento superficial



Jateamento com esferas